# Higher-order corrections to exclusive production of charmonia at $B$ factories

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As a test of the color-singlet mechanism of the nonrelativistic QCD (NRQCD) factorization approach, we consider the exclusive two-quarkonium productions in electron-positron annihilation $e^+e^- \to \eta_c + \gamma$ and $e^+e^- \to J/\psi + J/\psi$ at $B$ factories. The cross sections are computed to the next-to-leading order in $\alpha_s$ and are resummed to all orders in half the relative velocity $v$ of the charm quark in each meson rest frame. The available theoretical prediction of the cross section for $e^+e^- \to J/\psi + \eta_c$ at the same level of theoretical accuracies is consistent with the available experimental data. Those for $e^+e^- \to \eta_c + \gamma$ and $e^+e^- \to J/\psi + J/\psi$ that are computed new in this work can be tested against the data from future super $B$ factories.

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# 1   Introduction

Production of heavy quarkonium provides a unique opportunity to probe the framework of the nonrelativistic quantum chromodynamics (NRQCD) factorization approach [1], which is an effective field theory to describe production and decay of heavy quarkonium. In this approach, a production or decay rate of a heavy quarkonium is expressed as a linear combination of NRQCD long-distance matrix elements (LDMEs) and the short-distance coefficients of each LDMEs are insensitive to the long-distance nature of hadrons. Each LDME represents the transition rate of a heavy quark and antiquark ($Q\overline{Q}$) pair with a specific spectroscopic state to evolve into the physical quarkonium state. The NRQCD factorization for the decay is proved in Ref. [1]. While proofs of factorization for some of exclusive quarkonium production processes are available [2], the factorization in the inclusive quarkonium production is still a conjecture.

In order to predict the production cross section of a heavy quarkonium, it is required to determine all of the relevant NRQCD LDMEs. The universality of the LDMEs requires that the LDMEs are independent of specific processes. In addition, an LDME for a production is the same as that for the decay under the vacuum-saturation approximation of order $v^4$, where $v$ is half the relative velocity of the heavy quark in the quarkonium rest frame [1]. At present, the LDMEs are determined by comparing the NRQCD factorization formulas with the measured production and decay rates of a heavy quarkonium. The color-singlet LDMEs can be determined from the electromagnetic decays of a heavy quarkonium [3, 4]. In the case of the color-octet channels, they use the inclusive production rates of a heavy quarkonium at various colliders, Tevatron, LHC, HERA, $B$ factories, and so on [5]. Because the color-singlet channel also contributes to the inclusive production processes, the color-singlet LDMEs that are determined form the electromagnetic decays are used as input parameters.

Let us focus on the color-singlet LDMEs for the $S$-wave heavy quarkonia, in particular, $J/\psi$ and $\eta_c$ that depend dominantly only the color-singlet channel. The corresponding LDMEs are determined from $J/\psi \to \ell^+\ell^-$ and $\eta_c \to \gamma\gamma$ [3] and applied to various production processes involving $J/\psi$ or $\eta_c$. The cross section for $e^+e^- \to J/\psi + \eta_c$ at the $B$ factories has substantial contributions from the relativistic [6, 7] as well as QCD corrections [8]. The prediction [6, 7] that includes both relativistic and QCD corrections is consistent with the data [9, 10] within errors. Therefore, both relativistic and QCD corrections may have significant contributions in other exclusive $S$-wave quarkonium production in electron-positron annihilation. In this work, we compute the relativistic corrections, resummed to all orders in $v$, to the cross sections for the exclusive processes $e^+e^- \to \eta_c + \gamma$ and $e^+e^- \to J/\psi + J/\psi$ that have not been observed at the $B$ factories, yet. Then, we try to combine them together with the QCD corrections in order to present a more reliable prediction for

the cross section. The theoretical prediction can be tested against the data from future experiments at BELLE II or super $B$ factory.

## 2 Higher-order corrections

A schematic form of the differential cross section of a heavy quarkonium is $d\sigma \sim d\hat{\sigma}_n \langle O_n^H \rangle$, where $\langle O_n^H \rangle$ is the NRQCD LDME representing the transition of a $Q\overline{Q}$ pair with a spectroscopic state $n$ $(Q\overline{Q}_n)$ to a heavy quarkonium, $d\hat{\sigma}_n$ is the corresponding short-distance coefficient involving the production of the pair $Q\overline{Q}_n$ from the initial state, and the summation over $n$ is implicit. Because we consider only the color-singlet channels in this work, we use the corresponding LDME $\langle O_n \rangle_H$, that is accurately determined from the electromagnetic decay, for the decay by applying the vacuum-saturation approximation $\langle O_n^H \rangle \approx (2J+1)\langle O_n \rangle_H$, where $J$ is the total-angular-momentum quantum number of $H$.

The short-distance coefficient $d\hat{\sigma}_n$ is a perturbative series in the strong coupling $\alpha_s$. In various quarkonium production processes, $d\hat{\sigma}_n$ are known to the next-to-leading-order (NLO) accuracies in $\alpha_s$. The $K$ factor in each process depends on the choice of input variables like the heavy-quark mass $m_Q$ and the factorization scale $\mu$. Complete next-to-next-to-leading-order (NNLO) corrections in $\alpha_s$ are known only in electromagnetic decays of a heavy quarkonium, $J/\psi \to \ell^+\ell^-$ and $\eta_c \to \gamma\gamma$ [11] from which one can determine the color-singlet LDMEs. However, unless one computes the short-distance coefficients for another specific process to the same accuracies (NNLO in $\alpha_s$), the prediction may depend strongly on the factorization scale. Because the short-distance coefficients for $e^+e^- \to J/\psi + \eta_c$, $e^+e^- \to \eta_c + \gamma$, and $e^+e^- \to J/\psi + J/\psi$, that we consider in this work, have not been computed to NNLO in $\alpha_s$, we use the numerical values for the color-singlet NRQCD LDMEs that were determined from the electromagnetic decays at order $\alpha_s$.

At leading order (LO) in $\alpha_s$ the spin-triplet $S$-wave heavy-quarkonium production consists of QCD and QED processes. Among QED diagrams, the photon-fragmentation process in which a virtual photon fragments into the color-singlet spin-triplet $S$-wave $Q\overline{Q}$ pair, can be comparable to the QCD process. This enhancement happens when $M^* \gg m_H$, where $M^*$ is the typical virtuality of the internal lines other than the fragmenting photon and the quarkonium mass $m_H$ represents the virtuality of the fragmenting photon. In $e^+e^-$ annihilation $M^* \sim$ half the center-of-momentum energy $(\sqrt{s})$ and in hadron collisions $M^* \sim$ the transverse momentum $p_T$ of the quarkonium. In these limits, the enhancement due to the propagator denominator of the fragmenting photon overcomes the strong suppression factor $(\alpha/\alpha_s)^2$ of the QED process relative to the QCD process. In the $J/\psi$ production at hadron colliders, the QED contribution via photon fragmentation could be larger than that from the usual QCD process at sufficiently large $p_T$ [12]. In $e^+e^- \to J/\psi + \eta_c$ at $B$ factories, the

QED contribution through photon fragmentation can reach about $19\%$ of the QCD contribution at LO in $\alpha_s$ [6].

In addition, the order $\alpha_s v^2$ correction is also a potential source of large corrections. The corrections are available for $J/\psi \to \ell^+\ell^-$ [13], $B_c \to \ell\nu$ [14], $\eta_c \to \gamma\gamma$ and light hadrons [15], and $e^+e^- \to J/\psi + \eta_c$ [16]. In $J/\psi \to \ell^+\ell^-$ and $B_c \to \ell\nu$, the relativistic corrections at order $\alpha_s$ are resummed to all orders in $v$. In $S$-wave heavy quarkonium decays, the order $\alpha_s v^2$ corrections are not so large. For example, in the $J/\psi \to \ell^+\ell^-$ decay, the relativistic corrections at order $\alpha_s$ are at most $0.3\%$ [13]. In the case of $e^+e^- \to J/\psi + \eta_c$ at $B$ factories, the relativistic corrections at order $\alpha_s$ enhance the cross section mildly [16].

In the heavy-quarkonium process, both the QCD and relativistic corrections could be large. In this case, the interference between the amplitude at NLO in $\alpha_s$ and that for the relativistic corrections might also be large. For example, in the $e^+e^- \to J/\psi + \eta_c$ process, the $K$ factor from the QCD corrections is about 2 and the relativistic corrections to the short-distance coefficients increase the cross section by a factor of $40\%$ [6]. Then, the interference can reach about $26\%$ of the QCD process at LO in $\alpha_s$.

# 3    Relativistic corrections to an $S$-wave quarkonium

The order-$v^{2n}$ correction to $S$-wave quarkonium ($H$) process in the color-singlet channel is proportional to the ratio $\langle \boldsymbol{q}^{2n} \rangle_H$ of the LDME of relative order $v^{2n}$ to the LO one $\langle O_1 \rangle_H$ [3]. Here, $\boldsymbol{q}$ is the spatial component of half the relative momentum of the $Q$ and $\overline{Q}$ in the $Q\overline{Q}$ rest frame. According to Ref. [17] the amplitude $A[H]$ expanded to all orders in $v$ can be computed from the hard amplitude $T(\boldsymbol{q}^2)$ as

$$A[H] = \sum_n \left[ \frac{1}{n!} \left( \frac{\partial}{\partial \boldsymbol{q}^2} \right)^n T(\boldsymbol{q}^2) \right] \Bigg|_{\boldsymbol{q}^2=0} \langle \boldsymbol{q}^{2n} \rangle_H \langle O_1 \rangle_H^{1/2}. \tag{1}$$

If we make use of the relation $\langle \boldsymbol{q}^{2n} \rangle_H \approx \langle \boldsymbol{q}^2 \rangle_H^n$ [17], a generalized version of the Gremm-Kapustin relation [18], we can resum a class of relativistic corrections to all orders in $v$. Then, the expansion (1) is simplified as

$$A[H] = T\left( \langle \boldsymbol{q}^2 \rangle_H \right) \langle O_1 \rangle_H^{1/2}. \tag{2}$$

Because the relation $\langle \boldsymbol{q}^{2n} \rangle_H \approx \langle \boldsymbol{q}^2 \rangle_H^n$ [17] has errors of relative order $v^2$ due to the neglect of the spin-dependent potential and the gauge field contribution to the covariant derivative in the LDME in the Coulomb gauge [17], Eq. (2) is accurate to order $v^4$. However, the resummation of a class of relativistic corrections may still be useful in estimating the size of the complete relativistic corrections. If the resummed factorization formula involves uncomfortably large corrections, then one may cast doubt

on the convergence of the series. Once the resummed formula gives moderate corrections, then one may treat this as a clue that the series may converge well. Another strong point of this resummation method is that the computation is far easier than the fixed-order relativistic corrections that involve a large number of terms generated by the derivatives. The method is particularly effective in $e^+e^- \to J/\psi + J/\psi$ that we present later.

# 4  $e^+e^- \to J/\psi + \eta_c$

In this section, we consider the process $e^+e^- \to J/\psi + \eta_c$ at $B$ factories. This process proceeds through one photon exchange between leptonic and hadronic currents. In the early stage, there was a discrepancy by an order of magnitude between theoretical predictions and experiments for the cross section [9, 19]. Later, the discrepancy was resolved due to the improvements in both theory and experiment. The measured cross section has decreased compared to the first measurement [10] and the corresponding theoretical predictions were enhanced after including order-$\alpha_s$ and relativistic corrections [6, 7, 8]. The $K$ factor from NLO corrections in $\alpha_s$ is about 2, which depends on the charm-quark mass $m_c$ [8]. The pure relativistic corrections consist of direct and indirect contributions that come from the corrections to the short-distance coefficient and through the LDME that amount to $40\%$ and $72\%$ of LO contribution, respectively [6]. The interference between the QCD and relativistic corrections was also computed in Ref. [6]. Then, the total cross section is $17.6^{+7.8}_{-6.3}$ fb [6], which is consistent with the empirical data $25.6 \pm 4.4$ fb at Belle and $17.6 \pm 3.5$ fb at BABAR [10]. Recently, improvement to this process at $O(\alpha_s v^2)$ has been carried out, but the cross section is enhanced mildly [16].

The process $e^+e^- \to J/\psi + \eta_c$ has been proved to have a significant relativistic corrections in comparison with any other quarkonium process. Now we do not have serious discrepancy between theory and experiment regarding this process. However, there still remain some subtle issues. The current experimental data for $e^+e^- \to J/\psi + \eta_c$ come from $\mu^+\mu^- +$ at least two charged tracks, which account for the $\eta_c$ decay. If one further includes the events without charged tracks, then the measured cross section can be bigger than the current data. It is interesting to see if the uncalculated corrections at NNLO in $\alpha_s$ enhances the theoretical prediction to catch up with the possible experimental enhancement. If it is not the case, then the discrepancy between the theory and experiment may revive.

# 5  $e^+e^- \to \eta_c + \gamma$

In this section, we consider the process $e^+e^- \to \eta_c + \gamma$. This process proceeds through one photon exchange because the charge-conjugation parity in the final state is $-1$

[20]. Similarly, one may also consider the production of any heavy quarkonium with charge conjugation parity +1 associated with a photon. This process was suggested to be a good probe to the color-singlet mechanism of NRQCD, especially for $\eta_c(2S)$ [20]. Later, the NLO corrections in $\alpha_s$ and the relativistic corrections at order $v^2$ were computed [21].

At LO in $\alpha_s$ and $v$, the cross section for $e^+e^- \to \eta_c + \gamma$ at $\sqrt{s} = 10.58$ GeV is $82^{+21.4}_{-19.8}$ fb for $\langle O_1 \rangle_{\eta_c} = 0.437^{+0.111}_{-0.105}$ GeV$^3$ [20]. The NLO corrections in $\alpha_s$ decrease the cross section by 18 % and the relativistic corrections at order $v^2$ reduce the cross section by 12 % for $v^2 = 0.13$ [21]. However, this $v^2$ value is rather underestimated compared with the conventional value 0.3 for a charmonium. If one uses $v^2 = 0.23$ determined from the Cornell potential and resummed formula for the electromagnetic decay rates of $J/\psi$ and $\eta_c$ [3], then the order-$v^2$ corrections can reach 21 %. We carry out the resummation of relativistic corrections to all orders in $v$ and find that the cross section decreases by 17 %, which is slightly smaller than the $v^2$ corrections [22]. This implies that the $v^2$ expansion in this process converges rapidly. The relativistic corrections are comparable to the NLO corrections in $\alpha_s$. This indicates that the inclusion of both QCD and relativistic corrections may improve the predictive power of the theoretical prediction. Furthermore, it might be necessary to compute interference between the corrections of NLO in $\alpha_s$ and the relativistic corrections. Taking into account all of the corrections listed above, we find that $\sigma[e^+e^- \to \eta_c + \gamma] = 55.1$ fb for $\mu = 2m_c$ [22].

# 6  $e^+e^- \to J/\psi + J/\psi$

In this section, we consider the process $e^+e^- \to J/\psi + J/\psi$. This process proceeds through two photon exchange because both $J/\psi$ and $\gamma$ are in odd parities under charge conjugation. This process was originally suggested to resolve the $e^+e^- \to J/\psi + \eta_c$ puzzle [23]. However, the Belle Collaboration found no evidence of this process and set an upper bound for the cross section times the branching ratio of one of two $J/\psi$'s decaying into at least two charged particles to be 9.1 fb [10]. The angular distribution analysis of $e^+e^- \to J/\psi + \eta_c$ events by the Belle Collaboration also disfavored the double $J/\psi$ production at $B$ factories.

There are four Feynman diagrams in this process. Two of them are photon fragmentation diagrams, where each virtual photon evolves into a $J/\psi$. The remaining two are called the nonfragmentation diagrams. The photon fragmentation diagrams dominate over the nonfragmentation diagrams [24]. The nonfragmentation contribution is at most 0.1 % of the fragmentation contribution. The contributions of the interference between the fragmentation and nonfragmentation diagrams is about 13 % of the fragmentation contribution [23]. As is stated earlier, the dominance of the fragmentation contribution is due to the large enhancement factor from the propagator

denominator of the fragmenting photon whose virtuality is of order $m_{J/\psi}$. Because each contribution makes a separate gauge-invariant subset, gauge invariance remains although we use different strategies to compute each set of the amplitude. In Ref. [24], the authors employed the vector-meson-dominance (VMD) method for the photon fragmentation diagrams by replacing the photon-to-$J/\psi$ vertex by a coupling $g_{J/\psi}$, which is determined from the leptonic decay of $J/\psi$. This method includes automatically the relativistic and QCD corrections to the fragmentation contribution. In the case of the nonfragmentation diagrams, they used the standard NRQCD approach to compute the amplitude. Then, the total cross section for $e^+e^- \to J/\psi + J/\psi$ at $B$ factories is $1.69 \pm 0.35$ fb, which is well below the upper bound at Belle [24].

The computation of the NLO corrections in $\alpha_s$ was carried out within the framework of NRQCD [25]. The $K$ factor strongly depends on $m_c$ and the factorization scale $\mu$. For $\mu = 2m_c$, the $K$ factor is about 0.077 for $m_c = 1.5$ GeV and about 0.057 for $m_c = 1.4$ GeV, respectively [25]. We note that most of the NLO corrections in $\alpha_s$ come from the corrections to the photon fragmentation diagrams. If the VMD treatment is applied to the fragmentation diagrams, the $K$ factor can be large.

As we have mentioned, the resummation of relativistic corrections to all orders in $v$ is far easier than the computation of fixed-order relativistic corrections. This is particularly true in this case because of the $t$-channel electron propagator that is an additional source of relativistic corrections. We find that the resummation of relativistic corrections decreases the cross section by $58\,\%$ [22]. The sum of the relativistic corrections and the corrections of NLO in $\alpha_s$ can be even negative because both corrections are quite large. The inclusion of the interference between the relativistic and QCD NLO corrections may be helpful to cure the problem of the negative cross section because the interference is positive. Therefore, one must be very careful in combining these corrections, especially in the process $e^+e^- \to J/\psi + J/\psi$.

# 7  Summary

Exclusive heavy-quarkonium production in electron-positron collisions provides a unique opportunity to test the color-singlet mechanism of NRQCD. In this work, we have discussed three exclusive processes, $e^+e^- \to J/\psi + \eta_c$, $e^+e^- \to \eta_c + \gamma$, and $e^+e^- \to J/\psi + J/\psi$ at $B$ factories. The available theoretical prediction of the cross section for $e^+e^- \to J/\psi + \eta_c$ is consistent with the experimental values measured by the Belle and BABAR collaborations within uncertainties. We have computed the relativistic corrections to the cross sections for $e^+e^- \to \eta_c + \gamma$ and $e^+e^- \to J/\psi + J/\psi$ resummed to all orders in $v$. The predictions are further improved by adding additional corrections of NLO in $\alpha_s$. These new predictions can be tested against the data from future super $B$ factories.

## ACKNOWLEDGEMENTS

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